On the whole, the results of the investigation show that the proposed mathematical model of heat transfer is of sufficeint accuracy for engineering calculations, and may be used to determine the temperature fields and resulting heat fluxes in commercial furnaces.

NOTATION

x_0, y_0, z_0, ρ_0, φ_0, minimal abscissa, ordinate, z coordinate, radius vector, and polar angle of zone, m, rad; Δx, Δy, Δx, $\Delta\rho$, $\Delta\varphi$, dimensions of the zones along the coordinates, m, rad; y_E, z_E, ρ_E, corresponding coordinates of the surface passing through the tube axes, m; η_1, η_2, angles between the z axis and the surfaces lying close to and far from the z axis, rad; η_E, angle between the z axis and the surface passing through the tube axes, rad; z_{η_1}, z_{η_2}, $z_{\eta E}$, z coordinates of the points of intersection of the corresponding surfaces with the z axis, m; P_{ij}, radiation-transfer coefficients, W/K^4; Ω_{ij}, coefficients of convective—turbulent transfer, W/K; T_i, T_β, temperatures of zones i and β; K; $\delta^j{}_\gamma$, $\delta^j{}_\beta$, Kronecker deltas; c_j, free term of the j-th equation; N, total number of volume and surface zones.

LITERATURE CITED

1. V. G. Kashirskii, V. M. Sedelkin, and A. V. Paimov, "Zonal mathematical model of external heat transfer in tube furnaces," Izv. Vyssh. Uchebn. Zaved., Energ., No. 4, 91-96 (1977).
2. G. I. Marchuk (ed.), Monte Carlo Method in the Radiation-Transfer Problem [in Russian], Atomizdat, Moscow (1967).
3. H. C. Hottel and A. F. Sarofim, Radiative Transfer, McGraw-Hill, New York (1967).
4. V. M. Sedelkin, A. V. Paimov, V. A. Kamenskaya, and M. S. Ugol'nikov, "Method of computer calculation of the opticogeometric characteristics of reciprocal radiative transfer in screened furnaces taking account of nongray radiation of the volume and surface zones," in: Processes of Directed Heat Transfer [in Russian], Naukova Dumka, Kiev (1979), pp. 113-119.

NUMERICAL CALCULATION OF THE GENERALIZED ANGULAR EMISSION COEFFICIENTS IN TWO-DIMENSIONAL SYSTEMS

V. G. Lisienko and Yu. K. Malikov

UDC 536.3

We describe a method of calculating the generalized angular emission coefficients by expressing them in the form of finite series.

The calculation of the generalized angular emission coefficients is the most important step in applying zonal methods to the calculation of heat exchange in various radiating systems. The calculation of the generalized angular emission coefficients by direct integration is possible only in special cases [1]. The resulting formulas are complex and not very convenient in engineering applications. This deficiency is also present in the approximate method developed in [2, 3]. An exception is the approximate method of [4,5], in which relatively simple analytical expressions can be obtained with the help of theorems on the mean. In this method, the generalized angular emission coefficient is written as a product of a geometrical angular coefficient and a certain average transmissivity of the medium. The latter is considered as a purely geometrical characteristic of a radiating layer, and this leads to significant errors.

In the present paper, the generalized angular emission coefficients are obtained for a two-dimensional system as finite series, where each term is a product of a geometrical angular coefficient and the transmissivity of the medium. The resulting algorithm for the numerical computation of the generalized angular emission coefficients can be used to determine the coefficients for two-dimensional systems of complicated geometry. In this approach, the amount of calculation is about 100-1000 times less than in the Monte Carlo method [5] which is usually used in these problems.

S. M. Kirov Urals Polytechnical Institute, Sverdlovsk.

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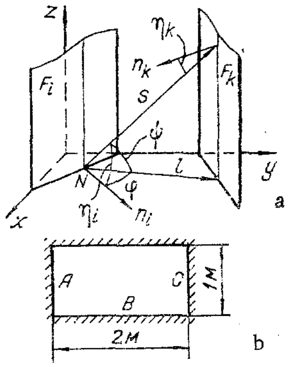

Fig. 1

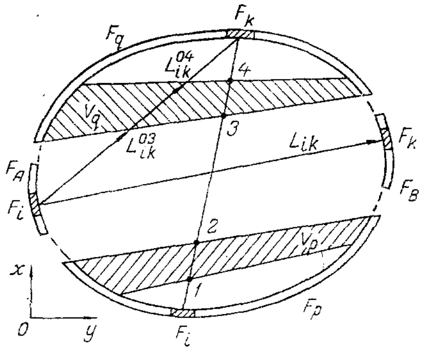

Fig. 2

Fig. 1. Cylindrical surface: a) pair of infinite strips; b) slot.

Fig. 2. Determination of the generalized angular emission coefficients.

It is well known that the generalized angular coefficients are defined in a similar way both for integral quantities in a series approximation formulation of the problem, and for the general case for the spectral quantities in each wavelength interval. Here for simplicity we will omit the index denoting the spectral interval.

We consider a pair of infinite strips along the z axis (Fig. 1a). The generalized angular coefficient from surface F_i onto surface F_k is defined by the relation [1]

$$\varphi_{ik}^{0}=\frac{1}{F_i}\int\limits_{(F_i)}\int\limits_{(F_k)}\frac{\cos\eta_i\cos\eta_k}{\pi s^2}\exp(-L(s))\,dF_i dF_k, \tag{1}$$

where $L(s)=\int\limits_0^s k(s)\,ds$ is the optical path of the ray.

We use a spherical coordinate system with origin at point N on the strip F_i, as shown in Fig. 1a. We assume that the attenuation coefficient of the medium does not depend on z. Then

$$L(s)=\int\limits_0^s k(x,\ y)\,ds=\frac{1}{\cos\psi}\int\limits_0^l k(x,\ y)\,dl=\frac{L_n}{\cos\psi}.$$

Integrating (1) with respect to the angle ψ using the above relation gives

$$\varphi_{ik}^{0}=\frac{1}{\pi F_i}\int\limits_{(F_i)}\int\limits_{\varphi_k(N)}M(L_n)\cos\varphi\,d\varphi\,dF_i, \tag{2}$$

where $\varphi_k(N)$ is the set of angles for which rays leaving point N are incident on F_k and

$$M(L)=\int\limits_{-\pi/2}^{\pi/2}\exp(-L/\cos\psi)\cos^2\psi\,d\psi.$$

The function M(L) was introduced and tabulated in [6] in the calculation of the generalized angular coefficients in cylindrical systems for a medium with a constant attenuation coefficient. The latter restriction is not essential because (2) is valid more generally (when the attenuation coefficient of the medium depends on x and y).

If the strips F_i and F_k are narrow enough, then L_n for all rays leaving F_i and incident on F_k will be approximately the same. Then the slowly varying function $M(L_n)$ can be pulled out of the integral sign, and (2) can be replaced with the approximate relation

$$\varphi_{ik}^{0}\approx\varphi_{ik}\frac{2}{\pi}M(L_{ik}). \tag{3}$$

We consider a pair of infinite cylindrical surfaces along the z axis whose cross sections in the xy plane are shown in Fig. 2. A medium with an attenuation coefficient dependent only on x and y occupies the space between the surfaces. We break up the surface F_A into M_A strips F_i of identical area and surface F_B into M_B strips F_k. Using (3) and the additivity of the angular emission coefficients, we obtain the approximate representation

$$\varphi_{AB}^{0}\approx\frac{2}{\pi M_A}\sum_{i=1}^{M_A}\sum_{k=1}^{M_B}\varphi_{ik}M(L_{ik}). \tag{4}$$

TABLE 1. Generalized Angular Emission Coefficients for a Slot (Fig. 1b)

Absorption coeff. of medium	Sum (4) for different values of $M_A = M_B$			Data of [8]
	3	5	10	
Generalized angular coeff. φ^0_{AB}				
0,05	0,3619	0,3630	0,3630	0,3630
0,1	0,3371	0,3448	0,3454	0,3455
0,5	0,2353	0,2401	0,2426	0,2434
1,0	0,1563	0,1645	0,1687	0,1704
Generalized angular coeff. φ^0_{AC}				
0,05	0,2078	0,2077	0,2077	0,2077
0,1	0,1895	0,1833	0,1831	0,1831
0,5	0,0701	0,0700	0,0700	0,0700
1,0	0,0222	0,0222	0,0222	0,0222

The coefficients φ^0_{AB} can be efficiently calculated according to (4) since the geometrical angular emission coefficients between arbitrary cylindrical surfaces can easily be found using the methods of [7] without numerical integration. Also the optical properties of the medium are, as a rule, slowly varying functions of the coordinates and this insures a rapid convergence to the series (4) for large M_A, M_B. For example, in Table 1 we give the results for the angular coefficients for a slot (see Fig. 1b) filled with an absorbing medium. Earlier these coefficients were obtained in [8] by numerical integration of (1) by the Gaussian method. It is clear that the series (4) converges very rapidly to the exact value, and accuracy sufficient for engineering applications can be obtained even when the surfaces are split into only three pieces.

We consider now the similar representation of the generalized angular coefficients φ^0_{Aq} between a surface F_A and a volume V_q (Fig. 2). Under the assumption that the optical properties of the gas in the region between the surfaces are constant (which is ordinarliy assumed in this method), we have [1]

$$\varphi^0_{Aq} = \frac{\alpha_q}{F_A} \int\limits_{(F_A)} \int\limits_{(V_q)} \frac{\cos\eta_A}{\pi s^2} \exp(-L(s))\, dV_q dF_A. \tag{5}$$

The integral over volume in (5) is written in spherical coordinates with the origin at the point $N \in F_k$. Then

$$dV_q = s^2 \cos\psi d\psi d\varphi ds, \quad \cos\eta_k = \cos\varphi\cos\psi,$$

where angles ψ and φ are defined as shown in Fig. 1. Assuming for simplicity that the surface bounding volume V_q is not concave, we obtain

$$\varphi^0_{Aq} = \frac{\alpha_q}{\pi F_A} \int\limits_{(F_A)} \int\limits_{\varphi_q(N)} \cos\varphi \int\limits_{-\pi/2}^{\pi/2} \cos^2\psi \int\limits_{s_3}^{s_4} \exp(-L(s))\, ds d\psi d\varphi dF_A, \tag{6}$$

where s_3 and s_4 are the path lengths of the ray from point N to its entrance into volume V_q, and from N to its exit from V_q, respectively, and $\varphi_q(N)$ is the set of azimuthal angles such that rays leaving point N are incident in volume V_q. The assumption that the optical properties of the gas are constant in the region under consideration means that the inner integral in (6) can be calculated along the ray path. Then the integration with respect to ψ can be carried out. The result is

$$\varphi^0_{Aq} = \frac{\alpha_q}{k_q} \frac{1}{\pi F_A} \int\limits_{(F_A)} \int\limits_{\varphi_q(N)} \cos\varphi\, (M(L_n^{03}) - M(L_n^{04}))\, dF_k \tag{7}$$

where $L_n^{0j} = \int\limits_0^{l_j} k(x, y)\, dl$, $j = 3, 4$, with l_3, l_4, the projections on the xy plane of ray paths up until entry into volume V_q and exit from it, respectively (Fig. 2).

The result (7) is exact. In doing the numerical calculations it is convenient to divide the surface F_A into M_A identical narrow strips F_k, and the boundary of the radiating volume F_q (see Fig. 2) into M_q strips F_i. Then the weakly varying function $M(L_n)$ can be taken out from under the integral sign and we obtain in place of (7) the approximate expression

$$\varphi^0_{Aq} \approx \frac{2\alpha_q}{\pi k_q M_A} \sum_{k=1}^{M_A} \sum_{i=1}^{M_q} \varphi_{ki} (M(L^{03}_{ki}) - M(L^{04}_{ki})). \tag{8}$$

The generalized angular emission coefficients φ°_{pq} from V_p to V_q (Fig. 2) can also be represented as a sum

$$\varphi^0_{pq} \approx \frac{\alpha_q}{2\pi k_q k_p V_p} \sum_{i=1}^{M_q} \sum_{k=1}^{M_p} F_k \varphi_{ki} [M(L^{14}_{ki}) + M(L^{23}_{ki}) - M(L^{13}_{ki}) - M(L^{24}_{ki})], \tag{9}$$

which rapidly converges to the exact value when we decrease the size of the strips F_i and F_k into which the surfaces F_q and F_p bounding the radiating volume are split up.

In practical applications of (4), (8), (9), it is necessary to calculate the function M(L), which in [6] is represented as an integral of modified Bessel functions. In engineering calculations, the following approximation can be used with an error not exceeding 0.007:

$$M(L) \approx 1.2852 \exp(-1.1L) + 0.2852 \exp(-2.053L).$$

The uniformity of relations (4), (8), (9) significantly simplifies the computer programing and leads to a decrease in the amount of computation necessary to determine the generalized angular emission coefficients. The error in the calculations can be estimated by comparing the results when different numbers of terms are left in the series. The algorithms based on (4), (8), (9) are exceptionally fast and adaptable, hence they can be used for calculating generalized angular emission coefficients in two-dimensional systems of complicated geometry.

NOTATION

φ°_{ik}, φ_{ik}, generalized and geometrical angular coefficients; k, α, effective attenuation and absorption coefficients of the medium; s, l, geometrical ray path and its projection on the xy plane; L(s), optical path of the ray; L_n, optical path of rays in the xy plane; L_{ik}, optical thickness of the medium between the centers of strips F_i and F_k in the xy plane; L^{mn}_{ik} (m, n = 0, 1, 2, 3, 4), the part of the optical path of rays (Fig. 2) propagating in the xy plane from strip F_i to F_k; M(L), Mikk function; F, surface; V, volume.

LITERATURE CITED

1. A. S. Nevskii, Radiative Heat Exchange in Ovens and Furnaces [in Russian], Metallurgiya, Moscow (1971).
2. S. P. Detkov, "Radiative transfer in two-dimensional problems," Inz.-Fiz. Zh., 10, No. 2, 245-251 (1966).
3. I. R. Mikk, "On the calculation of angular coefficients for a radiating system with an absorbing medium," Inz.-Fiz. Zh., 5, No. 11, 25-31 (1962).
4. E. S. Kiseleva and Yu. A. Surinov, "An engineering method of determining and calculating generalized angular emission coefficients" [in Russian], Tr. Moskovskogo Ekonomiko-Statisticheskogo Instituta, Fiz. Mat. Nauki (1969), pp. 97-104.
5. V. G. Lisienko, Increase of Heat Exchange in Ovens [in Russian], Metallurgiya, Moscow (1979).
6. I. R. Mikk and I. P. Épik, "Solution of some three-dimensional radiative heat exchange problems by reducing them to two-dimensional," Inzh.-Fiz. Zh., 4, No. 6, 90-100 (1961).
7. G. L. Polyak, "Algebra of radiative currents," Izv. Energeticheskogo Instituta im. G. M. Krzhizhanovskogo Akad. Nauk SSSR, 3, No. 1-2, 53-75 (1935).
8. S. P. Detkov and A. V. Vinogradov, "Generalized angular coefficients for a slot," Izv. Vyssh. Uchebn. Zaved., Energet., No. 10, 105-109 (1964).